University of Memphis
MATH 3242
Linear Algebra
Spring 2025
Dwiggins

**Homework Assignment # 2**

Chapter Three: Determinants

\# 6. $\begin{vmatrix} 5 & 0 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{vmatrix} = -5.$ The determinant of a triangular matrix is equal to the product of its diagonal elements.

\# 10. $\begin{vmatrix} -15 & 0 & 3 \\ 3 & 9 & -6 \\ 12 & -3 & 6 \end{vmatrix} = (3^3)\begin{vmatrix} -5 & 0 & 1 \\ 1 & 3 & -2 \\ 4 & -1 & 2 \end{vmatrix}$ (factoring out 3 from each of the three rows)

$= 27\left[ -5\begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} - 0 + 1\begin{vmatrix} 1 & 3 \\ 4 & -1 \end{vmatrix} \right]$ (expanding along the top row)

$= 27\,[\,-5(6-2) + (-1-12)\,] = 27(-20-13) = -891$ .

\# 12. $\begin{vmatrix} 2 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 4 & -1 & 3 & 0 \\ 5 & 2 & 1 & -1 \end{vmatrix} = -6.$ (again using the product of the diagonal elements for a triangular matrix.)

\# 20. $\begin{vmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 4 & -1 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 0 \\ 4 & 1 & -1 \end{vmatrix}$ (switching columns 2 and 3)

\# 22. $\begin{vmatrix} 1 & 3 & 1 \\ 0 & -1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 5 & 4 \\ 1 & 2 & 1 \end{vmatrix}$ (replace Row 2 with 2*Row1 + Row 2)

\# 30. $A = \begin{pmatrix} 10 & 2 \\ -2 & 7 \end{pmatrix}$ implies $\det(A) = 70 - (-4) = 74$.

We could use the shortcut method to find the inverse of A, but I'm pretty sure the textbook wanted me to use the method of row reduction to find $A^{-1}$, and that's what will be demonstrated on the next page.

(# 30 continued) We have $(A \mid I) = \left(\begin{array}{cc|cc} 10 & 2 & 1 & 0 \\ -2 & 7 & 0 & 1 \end{array}\right) \longleftrightarrow \left(\begin{array}{cc|cc} 1 & 0.2 & 0.1 & 0 \\ -1 & 3.5 & 0 & 0.5 \end{array}\right)$

$$\longleftrightarrow \left(\begin{array}{cc|cc} 1 & 0.2 & 0.1 & 0 \\ 0 & 3.7 & 0.1 & 0.5 \end{array}\right) \longleftrightarrow \left(\begin{array}{cc|cc} 1 & 0.2 & 0.1 & 0 \\ 0 & 1 & 1/37 & 5/37 \end{array}\right)$$

$$\longleftrightarrow \left(\begin{array}{cc|cc} 1 & 0 & 7/74 & -1/37 \\ 0 & 1 & 1/37 & 5/37 \end{array}\right) = (I \mid A^{-1}),$$ and so

$$\det(A^{-1}) = (1/37)^2 \begin{vmatrix} 3.5 & -1 \\ 1 & 5 \end{vmatrix} = (1/37)^2 (17.5 + 1) = (1/37)^2 (37/2) = 1/74 = 1/\det(A),$$

as required.

# 32. $(A \mid I) = \left(\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 8 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right) \longleftrightarrow \left(\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 6 & 12 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 \end{array}\right)$

$$\longleftrightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & -2 & -1 & 0 & 0 \\ 0 & 1 & 2 & 1/3 & 1/6 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array}\right) \longleftrightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2/3 & 1/6 & -1 \\ 0 & 0 & 1 & 1/2 & 0 & 1/2 \end{array}\right)$$

$$\longleftrightarrow (I \mid A^{-1}), \text{ with } A^{-1} = \begin{pmatrix} -2/3 & 1/6 & 0 \\ -2/3 & 1/6 & -1 \\ 1/2 & 0 & 1/2 \end{pmatrix},$$ and this gives

$$\det(A^{-1}) = (-2/3) \begin{vmatrix} 1/6 & -1 \\ 0 & 1/2 \end{vmatrix} - (1/6) \begin{vmatrix} -2/3 & -1 \\ 1/2 & 1/2 \end{vmatrix} + 0 = (-2/3)(1/12) - (1/6)(1/6) = -3/36$$

$= -1/12$, while $\det(A) = 2(-2-4) - 8(1-1) + 0$ (expanding using the third column of $A$), which gives $\det(A) = -12$, and so once again we have $\det(A^{-1}) = 1/\det(A)$.

# 40. The determinant of the coefficient matrix for the system $\left\{\begin{array}{l} 2x + 3y + z = 10 \\ 2x - 3y - 3z = 22 \\ 8x + 6y \qquad = -2 \end{array}\right\}$

is given by $\det(A) = 2\cdot3\cdot\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -3 \\ 4 & 2 & 0 \end{vmatrix}$ (after removing common factors from the first two columns)

$= 6 \cdot [1(2+4) - (-3)(2-4) + 0]$ (expanding along the third column)

$= 6 \cdot [6-6] = 0$. This means the given system has either no solution or infinitely many solutions. If we multiply the first equation by 3 and add that to the second equation, we get 8x + 6y = 52, which is inconsistent with the third equation, and so this system has no solution.

# 60. The triangular matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$ has determinant equal to –1, and

its adjoint is given by $\begin{pmatrix} -1 & -1 & -3 \\ 0 & -1 & -2 \\ 0 & 0 & -1 \end{pmatrix}$. Therefore, the inverse of $A$ is

is given by $-1\cdot\text{adj}(A) = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$. You can check this answer by

multiplying it by A and verifying that $A^{-1}A = I$.

# 64. The determinant for the system $\left\{\begin{array}{l} 4x_1 + 4x_2 + 4x_3 = 5 \\ 4x_1 - 2x_2 - 8x_3 = 1 \\ 8x_1 + 2x_2 - 4x_3 = 6 \end{array}\right\}$ is equal to zero, and so

this system does not have a unique solution. Performing the row reduction shows

$(A \mid b) \longleftrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 5/4 \\ 0 & 1 & 2 & 2/3 \\ 0 & 0 & 0 & 0 \end{array}\right)$, and so this system has infinitely many solutions.

One way to express the solution set is to set $x_1 = (19/12) - (1/2)x_2$ and $x_3 = (-1/3) - (1/2)x_2$ .