University of Memphis
MATH 3242
Linear Algebra
Spring 2025
Dwiggins

Homework Assignment # 1

Chapters One and Two

Chapter One Review, # 20. Solve the linear system $\left\{\begin{array}{l} 0.2x - 0.1y = 0.07 \\ 0.4x - 0.5y = -0.01 \end{array}\right\}$.

Multiply the first row by 20 and the second row by –10, giving the equivalent system $\left\{\begin{array}{l} 4x - 2y = 1.4 \\ -4x + 5y = 0.1 \end{array}\right\}$

Next, adding these two equations gives $3y = 1.5$, which means $y = 0.5$, and substituting this back into the first equation gives $4x - 1 = 1.4$, or $x = 0.6$. Checking this solution with the original system gives $(0.2)(0.6) - (0.1)(0.5) = 0.12 - 0.05 = 0.07$ and $(0.4)(0.6) - (0.5)(0.5) = 0.24 - 0.25 = -0.01$, as required.

32. The linear system $\left\{\begin{array}{l} 4x + 2y + z = 18 \\ 4x - 2y - 2z = 28 \\ 2x - 3y + 2z = -8 \end{array}\right\}$ is represented by the augmented matrix

$$\left(\begin{array}{ccc|c} 4 & 2 & 1 & 18 \\ 4 & -2 & -2 & 28 \\ 2 & -3 & 2 & -8 \end{array}\right).$$

Using the operations Row1 – Row 2 = New Row 2 and Row 1 – 2*Row 3 = New Row 3, this matrix is row-equivalent to

$$\left(\begin{array}{ccc|c} 4 & 2 & 1 & 18 \\ 0 & 4 & 3 & -10 \\ 0 & 8 & -3 & 34 \end{array}\right),$$

and then using 2*Row 2 – Row 3 = New Row 3 gives

$$\left(\begin{array}{ccc|c} 4 & 2 & 1 & 18 \\ 0 & 4 & 3 & -10 \\ 0 & 0 & 9 & -54 \end{array}\right).$$

Now performing the backwards substitution gives $z = -6$, $4y - 18 = -10$, or $y = 2$, and $4x + 4 - 6 = 18$, which gives $x = 5$.

Finally, checking this solution with the original system gives $20 + 4 - 6 = 18$, $20 - 4 + 12 = 28$, and $10 - 6 - 12 = -8$, as required.

48. After multiplying the second equation by –2 and adding it to the first equation in the linear system $\left\{\begin{array}{l} 2x_1 + 4x_2 - 7x_3 = 0 \\ x_1 - 3x_2 + 9x_3 = 0 \end{array}\right\}$, we obtain the equation $10x_2 - 25x_3 = 0$, or $x_3 = 0.4x_2$, and then substituting this back into the first equation gives $2x_1 + 4x_2 - 2.8x_2 = 0$, or $x_1 = -0.6x_2$. Thus, this system contains all the points on the line $2x_1 + 3x_3 = 0$ which lies in the plane $x_1 + x_2 - x_3 = 0$. (Note this contains the zero solution (0, 0, 0).) Another way to give the solution is to use the parametric equations $x_1 = -3t$, $x_2 = 5t$, $x_3 = 2t$.

Chapter One Review, # 66. Find a polynomial $p(x)$ which contains the points with coordinates $(-1, -1)$, $(0, 0)$, $(1, 1)$, and $(2, 4)$. It suffices to assume this polynomial has degree three or less, and so we assume it has the form $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$. Substituting in the value $p(0) = 0$ automatically gives $a_0 = 0$, and so we have $p(x) = a_1x + a_2x^2 + a_3x^3$. Next, using $p(-1) = -1$ and $p(1) = 1$ gives the linear system $\left\{ \begin{array}{c} -a_1 + a_2 - a_3 = -1 \\ a_1 + a_2 + a_3 = 1 \end{array} \right\}$. Adding these two equations shows $a_2 = 0$, and so we have $p(x) = a_1x + a_3x^3$. Now we can use $p(1) = 1$ and $p(2) = 4$ to obtain the system $\left\{ \begin{array}{c} a_1 + a_3 = 1 \\ 2a_1 + 8a_3 = 4 \end{array} \right\}$. Solving this system gives $a_1 = 2/3$ and $a_3 = 1/3$, and so we have $p(x) = (2x + x^3)/3$ as the solution to this exercise.

The graph of this polynomial goes through the given points, including (0, 0). This function is always strictly increasing, with the inflection point at (0, 0).

72. The electric circuit illustrated in the textbook consists of two loops (top and bottom), each with their own voltage source, as well as two branch points on the left and the right. Based on the orientation of the electrodes in the two voltage sources, I have assumed that the current I_1, in the top loop, goes through the resistor R_1 in the direction towards the left, with the same orientation for the current I_3, in the bottom loop, through the resistor R_3. For the shunt current in the middle, I have assumed the current I_2 goes through the resistor R_2 in the direction towards the right. (I will be proven wrong if I get a negative value for I_2.)

Combining Ohm's Law ($V = IR$) with Kirchhoff's Rules, the top loop gives the equation $I_1R_1 + I_2R_2 = 3$ volts, or $3I_1 + 4I_2 = 3$, while the bottom loop gives $2I_3 + 4I_2 = 2$. For the branch points, Kirchhoff's Rules state the total incoming current must equal the outgoing current, and both branches give the same equation, $I_1 + I_3 = I_2$. Substituting this into the two loop equations gives the system $\left\{ \begin{array}{c} 7I_1 + 4I_3 = 3 \\ 4I_1 + 6I_3 = 2 \end{array} \right\}$. Finally, solving this system gives the solution to this circuit problem, with the three currents given by

$I_1 = 5/13$ (amperes), $I_2 = 6/13$, and $I_3 = 1/13$.

Chapter Two Review, # 4. Using the definition of matrix multiplication, formed by inner products between row vectors and column vectors, we have

$$\begin{pmatrix} 1 & 5 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} 6 & -2 & 8 \\ 4 & 0 & 0 \end{pmatrix} = \begin{pmatrix} (1)(6) + (5)(4) & (1)(-2) + (5)(0) & (1)(8) + (5)(0) \\ (2)(6) + (-4)(4) & (2)(-2) + (-4)(0) & (2)(8) + (-4)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 26 & -2 & 8 \\ -4 & -4 & 16 \end{pmatrix}.$$

8. The linear system $\left\{ \begin{array}{l} 2x_1 - x_2 = 5 \\ 3x_1 + 2x_2 = -4 \end{array} \right\}$ has as its matrix equation $Ax = b$, with

$A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ and $b = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$, giving $(A \mid b) = \left(\begin{array}{cc|c} 2 & -1 & 5 \\ 3 & 2 & -4 \end{array}\right)$, which

is row-equivalent to $\left(\begin{array}{cc|c} 2 & -1 & 5 \\ 7 & 0 & 6 \end{array}\right)$, using 2*Row 1 + Row 2 = New Row 2.

Thus, we have $x_1 = 6/7$, and so $2x_1 - x_2 = 12/7 - x_2 = 5$ gives $x_2 = -23/7$.

Check: $2x_1 - x_2 = 12/7 + 23/7 = 35/7 = 5$, and
$3x_1 + 2x_2 = 18/7 - 46/7 = -28/7 = -4$, as required.

10. $(A \mid b) = \left(\begin{array}{ccc|c} 2 & 3 & 1 & 10 \\ 2 & -3 & -3 & 22 \\ 4 & -2 & 3 & -2 \end{array}\right) \leftrightarrow \left(\begin{array}{ccc|c} 2 & 3 & 1 & 10 \\ 0 & 6 & 4 & -12 \\ 0 & 8 & -1 & 22 \end{array}\right)$ using R1 – R2 and 2*R1 – R3

$$\leftrightarrow \left(\begin{array}{ccc|c} 2 & 3 & 1 & 10 \\ 0 & 1.5 & 1 & -3 \\ 0 & 8 & -1 & 22 \end{array}\right) \leftrightarrow \left(\begin{array}{ccc|c} 2 & 3 & 1 & 10 \\ 0 & 1.5 & 1 & -3 \\ 0 & 9.5 & 0 & 19 \end{array}\right)$$

This gives $9.5x_2 = 19$, or $x_2 = 2$, and so the second row gives $1.5x_2 + x_3 = 3 + x_3 = -3$, which means $x_3 = -6$. Finally, the first row gives $2x_1 + 6 - 6 = 10$, or $x_1 = 5$. Checking these results against the original system gives $2x_1 + 3x_2 + x_3 = 10 + 6 - 6 = 10$, $2x_1 - 3x_2 - 3x_3 = 10 - 6 + 18 = 22$, and $4x_1 - 2x_2 + 3x_3 = 20 - 4 - 18 = -2$, as required.

12. $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$, $A^T = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$, $A^TA = \begin{pmatrix} 13 & -3 \\ -3 & 1 \end{pmatrix}$, $AA^T = \begin{pmatrix} 10 & 6 \\ 6 & 4 \end{pmatrix}$

Note that while A^TA and AA^T are not the same matrix (this commutative property is true for inverses, but not transposes), it is true that each of these products is equal to its own transpose.

14. $A = (1, -2, -3)$ is a 1 x 3 row vector, and $A^T = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$ is a 3 x 1 column vector.

This means $A^TA = \begin{pmatrix} 1 & -2 & -3 \\ -2 & 4 & 6 \\ -3 & 6 & 9 \end{pmatrix}$ is a 3 x 3 matrix, while

$AA^T = (1)(1) + (-2)(-2) + (-3)(-3) = 1 + 4 + 9 = 14$ is a scalar (1 x 1).

18. To find the multiplicative inverse of $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$, use row operations on

$$(A \mid I) = \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$ using –R3 + R2 and –R3 + R1

$$\leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) = (I \mid A^{-1}), \text{ and so } A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

This result may be verified by using matrix multiplication to show $A^{-1}A = I$.

20. To solve the system $\left\{ \begin{array}{c} 3x_1 + 2x_2 = 1 \\ x_1 + 4x_2 = -3 \end{array} \right\}$, represented by the matrix equation $AX = b$,

use the 2 x 2 inverse matrix $A^{-1} = (1/10)\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}$. (The fraction in front comes from

det(A)= (3)(4) – (2)(1) = 12 – 2 = 10.) Then the solution to the system is given by

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A^{-1}b = (0.1)\begin{pmatrix} 4 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = (0.1)\begin{pmatrix} 4+6 \\ -1-9 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Check: $3x_1 + 2x_2 = 3 - 2 = 1$ and $x_1 + 4x_2 = 1 - 4 = -3$, as required.

22. $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \Rightarrow (A \mid I) = \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$ use R1 – R2 and 2R1 – R3

$$\leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 & -1 & 0 \\ 0 & 1 & 3 & 2 & 0 & -1 \end{array}\right) \leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 & -1 & 0 \\ 0 & -2 & -6 & -4 & 0 & 2 \end{array}\right) \leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 & -1 & 0 \\ 0 & 0 & -5 & -3 & -1 & 2 \end{array}\right)$$

$$\leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & -1 \\ 0 & 2 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0.6 & 0.2 & -0.4 \end{array}\right) \leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -0.2 & -0.4 & 0.8 \\ 0 & 2 & 0 & 0.4 & -1.2 & 0.4 \\ 0 & 0 & 1 & 0.6 & 0.2 & -0.4 \end{array}\right) \leftrightarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -0.4 & 0.2 & 0.6 \\ 0 & 1 & 0 & 0.2 & -0.6 & 0.2 \\ 0 & 0 & 1 & 0.6 & 0.2 & -0.4 \end{array}\right) = (I \mid A^{-1}) \Rightarrow X = A^{-1}b = (0.2)\begin{pmatrix} -2 & 1 & 3 \\ 1 & -3 & 1 \\ 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$